STRESS INTENSITY FACTORS OF EDGE CRACKS IN DISSIMILAR JOINT PLATES

Nik Ainun Binti Nik Ismail

A thesis submitted in fulfillment of the requirement for the award of the Degree of Master of Mechanical Engineering

FACULTY OF MECHANICAL AND MANUFACTURING ENGINEERING
UNIVERSITI TUN HUSSEIN ONN MALAYSIA

JUNE 2014
ABSTRACT

Nowadays, there are many applications in which need the combination or different materials. The development of this is caused by the mechanical wear problem, a high temperature situation or other conditions in which different properties are required from different parts of the same applications. This problem brings about the need for joining dissimilar materials. However the combination process between two dissimilar materials can caused mechanical mismatch and may lead to catastrophic failure or crack. Hence, this study will focused on the stress intensity factor on edge crack between dissimilar joint plates. The study focused on the investigation of edge crack behavior of dissimilar joint plates and to find out stress intensity factor (SIF) of dissimilar joint plates under different conditions by using finite element software. In this research, the investigation simulation are conducted by using finite element analysis software, ANSYS. A program that consist a coding of joining dissimilar material with centre and offset edge crack have been developed using ANSYS software. Data of stress intensity factor, $K$ produced by ANSYS software then transform to dimensionless stress intensity factor, $F$. Relationship between mechanical mismatch, $a$, ratio of stress, $\beta$, relative crack depth, $a/w$ and relative offset distance, $b/h$ to the dimensionless SIF, $F$ are analyze and discussed.
PADA MASA KINI, TERDAPAT Banyak aplikasi yang memerlukan gabungan 2 atau lebih bahan-bahan yang berbeza. Perkembangan ini disebabkan oleh masalah mekanikal bahan, keadaan suhu yang tinggi atau keadaan lain di mana sifat yang berbeza diperlukan dari bahagian yang berlainan pada aplikasi yang sama. Masalah ini telah membawa kepada keperluan bagi menyambungkan dua bahan yang berbeza dalam satu aplikasi yang sama. Walau bagaimanapun proses gabungan antara dua bahan yang tidak serupa boleh menyebabkan terjadinya ketidak padanan mekanikal dan boleh menyebabkan kegagalan bencana atau retak. Oleh itu, kajian ini akan memberi tumpuan kepada faktor keamatan tekanan pada retak di antara dua gabungan bahan yang berbeza. Kajian ini memberi tumpuan kepada kelakuhan retak tepi kepada gabungan bahan yang berbeza seterusnya mencari nilai faktor keamatan tekanan yang mungkin terjadi kepada gabungan bahan berbeza ini dengan kehadiran pelbagai parameter berbeza. Dalam kajian ini, simulasi ujian dibuat dengan menggunakan perisian ANSYS. Satu program yang mengandungi koding bagi menghasilkan gabungan bahan berbeza dengan kehadiran retakan di tepi telah dibangunkan menggunakan perisian ANSYS. Data faktor keamatan tekanan, \( K \) yang dihasilkan oleh perisian ANSYS kemudian diubah kepada faktor keamatan tekanan tanpa dimensi, \( F \). Seterusnya, hubungan antara sifat mekanikal berbeza, \( \alpha \), nisbah tekanan, \( \beta \), nisbah kedalaman retak, \( a/w \) dan nisbah jarak retak, \( b/h \), kepada faktor keamatan tekanan tanpa dimensi, \( F \) dianalisis dan dibincangkan.
## CONTENTS

**TITLE**

1

**DECLARATION**

2

**DEDICATION**

3

**ACKNOWLEDGEMENT**

4

**ABSTRACT**

5

**ABSTRAK**

6

**CONTENTS**

7

**LIST OF FIGURES**

8

**LIST OF TABLES**

9

**LIST OF SYMBOLS**

10

### CHAPTER 1 INTRODUCTION

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Background of study</td>
<td>1</td>
</tr>
<tr>
<td>1.2</td>
<td>Problem Statement</td>
<td>2</td>
</tr>
<tr>
<td>1.3</td>
<td>Project objectives</td>
<td>3</td>
</tr>
<tr>
<td>1.4</td>
<td>Project scopes</td>
<td>3</td>
</tr>
<tr>
<td>1.5</td>
<td>Summary</td>
<td></td>
</tr>
</tbody>
</table>
4.4.3 Effect of relative crack depth, $a/w$ and relative offset distance, $b/h$ to the value of dimensionless SIF, $F$

CHAPTER 5 CONCLUSION 57
5.1 Overview 57
5.2 Conclusion 57

REFERENCES 59

APPENDIXES 62
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Modes of crack</td>
<td>8</td>
</tr>
<tr>
<td>2.2</td>
<td>Effect of thickness on $K_C$</td>
<td>10</td>
</tr>
<tr>
<td>2.3</td>
<td>Failure stress against fracture toughness graph</td>
<td>10</td>
</tr>
<tr>
<td>2.4</td>
<td>Coordinate system around an interface crack</td>
<td>12</td>
</tr>
<tr>
<td>2.5</td>
<td>Interface crack at bi material specimen</td>
<td>12</td>
</tr>
<tr>
<td>2.6</td>
<td>Geometrical configuration of bi-material bonded plate</td>
<td>13</td>
</tr>
<tr>
<td>2.7</td>
<td>The edge interface cracks in a bonded strip</td>
<td>13</td>
</tr>
<tr>
<td>2.8</td>
<td>Constant value of C1 and C2 for various combinations of material</td>
<td>15</td>
</tr>
<tr>
<td>2.9</td>
<td>Distribution of stress intensity factors</td>
<td>18</td>
</tr>
<tr>
<td>2.10</td>
<td>FEM model of meshes near crack tip</td>
<td>19</td>
</tr>
<tr>
<td>2.11</td>
<td>Step of process in numerical simulation</td>
<td>20</td>
</tr>
<tr>
<td>2.12</td>
<td>Example of ANSYS capabilities</td>
<td>22</td>
</tr>
<tr>
<td>3.1</td>
<td>Flowchart of the structural analysis by ANSYS</td>
<td>24</td>
</tr>
<tr>
<td>3.2</td>
<td>Geometrical configurations of parts</td>
<td>25</td>
</tr>
<tr>
<td>3.3</td>
<td>Edge crack on the joining line of material</td>
<td>26</td>
</tr>
<tr>
<td>3.4</td>
<td>Edge crack offset to material 1</td>
<td>26</td>
</tr>
<tr>
<td>3.5</td>
<td>Edge crack offset to material 2</td>
<td>27</td>
</tr>
<tr>
<td>3.6</td>
<td>Difference between exact analysis and FEM</td>
<td>27</td>
</tr>
<tr>
<td>3.7</td>
<td>Library of element type</td>
<td>28</td>
</tr>
<tr>
<td>3.8</td>
<td>Define material model behavior</td>
<td>28</td>
</tr>
<tr>
<td>3.9</td>
<td>Examples of nodes</td>
<td>29</td>
</tr>
<tr>
<td>3.10</td>
<td>Create key point in active coordinate system</td>
<td>29</td>
</tr>
<tr>
<td>3.11</td>
<td>Meshing element from ANSYS</td>
<td>30</td>
</tr>
<tr>
<td>3.12</td>
<td>Result of stress intensity factor</td>
<td>31</td>
</tr>
</tbody>
</table>
3.13 Simulation parts of centre edge crack between dissimilar material
3.14 Simulation parts of offset edge crack between dissimilar material
4.1 The comparison of verification data a) $\alpha = 2$, b) $\alpha = 4$ and c) $\alpha = 10$
4.2 Plotted graph of centre edge crack simulation (a) subjected to in plane tension, (b) subjected to bending moment, (c) subjected to both cases
4.3 Dimensionless SIF, $F$ versus relative crack depth, $a/w$, (a) upper cracks, (b) lower cracks and (c) SIF ratio between upper and lower cracks.
4.4 Dimensionless SIF, $F$ versus relative crack depth, $a/w$, (a) upper cracks, (b) lower cracks (c) combined cases upper and lower crack (d) SIF ratio between upper and lower cracks.
4.5 Dimensionless SIF, $F$ versus relative crack depth, $a/w$, (a) upper cracks, (b) lower cracks (c) SIF ratio between upper and lower cracks.
4.6 Dimensionless SIF, $F$ versus relative crack depth, $a/w$, (a) upper cracks, (b) lower cracks (c) SIF ratio between upper and lower cracks.
4.7 Dimensionless SIF, $F$ versus relative crack depth, $a/w$, (a) upper cracks, (b) lower cracks (c) combined cases upper and lower crack (d) SIF ratio between upper and lower cracks.
LIST OF TABLES

2.1 Order of stress singularity for various combinations of materials 14
2.2 Values of $C_1$ 16
2.3 Values of $C_2$ 17
2.4 Stress intensity factors across thickness 18
3.1 Simulation condition for verification analysis of edge crack 32
3.2 Result for $F$ of an edge crack between rectangular dissimilar material by Toshiro et al (2000) 32
3.3 Condition for simulation of centre edge crack between dissimilar material 33
3.4 Condition for simulation of offset edge crack between dissimilar material 35
4.1 Result of verification process 37
4.2 Result of center edge crack simulation with value of $\alpha$ is 1,2,3 and 4 39
4.3 Offset edge crack simulation data 41
4.4 The Dimensionless SIF value for upper material crack, $F_1$ of offset edge crack for $\beta = 1.0$ and $\alpha$ value 0.3, 1.0 and 3.0 42
4.5 The Dimensionless SIF value for lower material crack, $F_2$ of offset edge crack for $\beta = 1.0$ and $\alpha$ value 0.3, 1.0 and 3.0 43
4.6 The Dimensionless SIF value for upper material crack, $F_1$ of offset edge crack for different $\beta$, $\alpha$ and $a/w$ value 45
4.7 The Dimensionless SIF value for lower material
crack, \( F_2 \) of offset edge crack for different \( \beta, \alpha \)
and \( a/w \) value

4.8 The Dimensionless SIF value for upper material
crack, \( F_1 \) of offset edge crack for \( \beta = 0.5 \) and
\( \alpha = 1 \)

4.9 The Dimensionless SIF value for lower material
crack, \( F_2 \) of offset edge crack for \( \beta = 0.5 \) and
\( \alpha = 1 \)

4.10 The Dimensionless SIF value for \( F_1/F_2 \) of offset
edge crack for \( \beta = 0.5 \) and \( \alpha = 1 \)

4.11 The Dimensionless SIF value for upper material
crack, \( F_1 \) of offset edge crack for \( \beta = 0.5 \) and
\( \alpha = 3 \)

4.12 The Dimensionless SIF value for lower material
crack, \( F_2 \) of offset edge crack for \( \beta = 0.5 \) and
\( \alpha = 3 \)

4.13 The Dimensionless SIF value for \( F_1/F_2 \) of offset
edge crack for \( \beta = 0.5 \) and \( \alpha = 3 \)

4.14 The Dimensionless SIF value for upper material
crack, \( F_1 \) of offset edge crack for \( \beta = 0.5 \) and
\( \alpha = 5 \)

4.15 The Dimensionless SIF value for lower material
crack, \( F_2 \) of offset edge crack for \( \beta = 0.5 \) and
\( \alpha = 5 \)

4.16 The Dimensionless SIF value for \( F_1/F_2 \) of offset
edge crack for \( \beta = 0.5 \) and \( \alpha = 5 \)
LIST OF SYMBOLS

\begin{itemize}
  \item $K$: Stress intensity factor
  \item $E$: Young modulus
  \item $v$: Poison ratio
  \item $a$: Length of crack
  \item $w$: Width of model
  \item $b$: Interval of crack
  \item $h$: Height of model
  \item $K_c$: Fracture toughness
  \item $\sigma$: Tensile stress
  \item $F$: Dimensionless stress intensity factor
  \item $t$: Thickness
  \item $M$: Moment
  \item $\alpha$: Mechanical mismatch
  \item $\beta$: Ratio of stress
  \item $a/w$: Relative crack depth
  \item $b/h$: Relative offset distance
\end{itemize}
CHAPTER 1

INTRODUCTION

1.1 Background of study

Nowadays, there are many applications in which need the combination or different materials. The development of this is caused by the mechanical wear problem, a high temperature situation or other conditions in which different properties are required from different parts of the same applications. This problem brings about the need for joining dissimilar materials. However the combination process between two dissimilar materials can caused mechanical mismatch and may lead to catastrophic failure or crack.

Failure or crack is a conditions in which solid materials fail under the action of external loads. Crack has seemed like a main phenomenon in mechanics of materials. Crack can cause failure of a component especially on a joining and assembly process. Failure of materials will cause huge cost to the industries. What is more worrying is the failure or crack can lead to the accidents involving human life. Because of this, field known as fracture mechanics have been introduced to overcome this problem.

For the past 50 years, fracture mechanics have been introduced in accordance to the crack studies. Fracture mechanics methodology is based on the assumption that all engineering materials contain cracks from which failure starts. The estimation of the remaining life of machine or structural components requires knowledge of the redistribution of stresses caused by the introduction of cracks in conjunction with a crack growth condition. Cracks result in high stress elevation in
the neighborhood of the crack tip, which should receive particular attention since it is at that point that further crack growth takes place.

Cracks can be classified according to various criteria. First criteria is the origin of the crack. One need to classified either the cracks are due to shrinkage and temperature variations in restrained elements or due to load producing local tension. Other than this, crack also can be classified in accordance to its shape and pattern either it is a single crack, multiple crack or branching cracks. Third criteria is the position of the cracks. In general, there are three type of crack position which are the centre cracks, single edge crack or multiple edge cracks. Last criteria for classification of crack is the crack deformation modes which have four modes namely opening mode (mode I), sliding mode (mode II), tearing mode (mode III) and last mode which is the mixed mode.

1.2 Problem statement

The failure of cracked components is governed by the stresses in the vicinity of the crack tip. The singular stress contribution is characterized by the stress intensity factor, $K$. Stress intensity factors or also known as driving force for fracture is dependent on the geometry of the component and on the special loading conditions (tension, bending, thermal stresses, etc).

As the stress intensity factors is one of the main problem in studying the propagation of crack, this project focus on the study of stress intensity factors of offset edge cracks in dissimilar joint plates. Currently there is limited stress intensity factor for offset edge crack in literature especially for cracks occurred in dissimilar joint plates. Therefore this study focus on the stress intensity factors for offset edge cracks in the dissimilar joint plates under tension and bending loadings.
1.3 **Project objectives**

Based on the problem statement, there are two objectives for this study which are:

i. To investigate edge crack behavior of dissimilar joint plates using finite element method

ii. To find out stress intensity factor (SIF) of dissimilar joint plates under different conditions using ANSYS Software

1.4 **Project scope**

This study covers the edge crack modeling using ANSYS for finite element analysis. The scope for study are:

i. Each analysis involves two types of material with fixed values of modulus elasticity, $E$ (200 GPa) for material one.

ii. Two materials with different mechanical properties are joined with an assumption that both materials are elastic.

iii. The cracks are located at the edge of dissimilar joint plates. Two conditions of cracked are assumed which are at the centre of the dissimilar joint plates and offset cracks.

iv. Stress intensity factors result obtained by changing data of young modulus, $E$, for material 2, ratio of $a/w$, $b/h$ and ratio of the pressure from tension loading to pressure from bending loading, $\beta$.

v. The dimensionless stress intensity factors, $F$ at the crack tips are calculated and discussed.

1.5 **Summary**

Rapid development in the field of manufacturing has seen many improvements have been made to improve the quality of human life. This includes the usage of several manufacturing processes which allows the joining process of two different materials in order to get better quality of the product. However, theoretically combination of materials usually will be exposed to some continuous stress that allow some crack in the
joining area. This crack if not treated well will propagate and cause a very big impact to the human life.

To overcome this problem, the understanding of fracture mechanics, fundamental of fatigue and finite element method is important to ensure the successful for this research. The nature of crack tip core regions and stress intensity factors are important factors in understanding fracture mechanics. An assumption of two dimensional plane stress or plane strain delivers useful two dimensional results with reasonable accuracy. This research depends on the theory value of the stress intensity factors and verified by using ANSYS software for finite element analysis.
CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter provides a comprehensive review related to the topic contained in this study. It explained on the concept of stress intensity factors in conjunction to the growth of the crack. In conclusion, this chapter explained further on the concept behind the crack initiation and crack growth related to the stress intensity factors by using numerical method which in this research by using ANSYS software.

2.2 Fracture mechanics

According to Gopichand et al (2012) fracture mechanics is a field of solid mechanics that deal with the mechanical behaviour of cracked bodies. Fracture is a problem that society has faced for as long as there have been man-made structures.

Barsom & Rolfe (1999) in his book explained that fracture mechanics is a method of characterizing the fracture behaviour of sharply notched structural members (cracked or flawed) in terms that can be used directly by the engineer. Fracture mechanics is based on a stress analysis in the vicinity of a notch or crack. It can also be used to predict the crack approach a critical size in fatigue or by environmental influences. The fracture mechanics approach have three important variables which are:
i. Fracture toughness of the material
ii. Applied stress
iii. Flaw size

Based on Anderson (2005) there are two alternatives approaches for fracture analysis. There are:

i. Stress intensity approach
   Each stress component is proportional to a single constant, \( K_I \). This constant is called stress intensity factor. It is completely characterizes the crack tip conditions in a linear elastic material. The formula for stress intensity factor is given by,
   \[
   K_I = F \sigma \sqrt{\pi a}
   \]  
   \[\text{(2.1)}\]

ii. Energy criterion
   The energy approach states that crack extension occurs when the energy available for crack growth is sufficient to overcome the resistance of the material. The materials resistance may include the surface energy, plastic work or other type of energy dissipation associated with a propagating crack.

2.2.1 Fracture process

Generally, fracture process occurs in a material in four steps as explained by Naman (2012). The steps are described below:

i. The first step is local yielding in the vicinity of defects or material and geometric singularities. The degree of singularity has a major influence on the magnitude of the plastic zone and the stress concentration. In repeated loading, there is hardening, which raises the yield stress, \( \sigma_y \). The material located near the notch tip becomes very strong, resulting in the creation of a first crack.

ii. Second step is the formation of cracks. This step can be due to surface treatments, with the treatment or thermal loading generating residual stresses
well above the yield strength. The material may also have cracks from static or variable mechanical loading.

iii. The third step is the real beginning of cracks. This propagation can be sudden or successive. Often there is successive propagation with the size of the crack increasing until it reaches a critical size, causing sudden propagation.

iv. The final step is the sudden propagation. It may be accompanied by generalized large strain (necking) or can occur without significant strain for brittle fracture.

### 2.2.2 Fracture modes

From a macroscopic point of view, they are two main types of fracture which is plane fracture and inclines fracture. Plane fracture corresponds to a flat fracture surface that is generally perpendicular to the direction of maximum principal stress. While inclined fracture presents a crack angle in the direction transverse to the direction of propagation. It is often accompanied by large strains.

For a plate with a through thickness crack, the loading on the crack is typically described as one of three types or modes. The modes of crack is described as below:

i. Mode I : Crack opening mode, where the displacements at the lips of the crack are perpendicular to the direction of propagation.

ii. Mode II : In plane shear mode, where the displacements at the lips of the crack are parallel to the direction of propagation.

iii. Mode III : Out of plane shear mode, where the displacements at the lips of the crack are parallel to the toe of the crack.

In addition, the crack may be simultaneously subjected to a combination of these loading modes, known as mixed mode loading.
2.2.3 Elementary fracture mechanics

The geometries of cracks, with radius of curvatures approaching zero at the crack tip cause stress fields that approach infinity proportional to the reciprocal of the square root of the distance from the crack tip (Byskov, 1984). This occurs even at low load levels. As such, commonly used failure measures such as Von Mises are not applicable (Shukla & Dally, 2010). As stated by Xian-Kui & Joyce (2012), the stress intensity factor, $K$ or also known as SIF was first proposed by Irwin (1957) and can be thought of as a measure of the effective local stress at the crack tip. An increasing stress intensity factor, $K$ indicates the stress near the crack tip is increasing. With this linear elastic fracture mechanics approach of characterizing the crack tip stresses, small amounts of plasticity may be viewed as taking place within the crack tip stress field and neglected for the characterization (Paris & Sih, 1965). Stress intensity factor, $K$ is designated by the mode of loading, such as $K_I$, $K_{II}$ and $K_{III}$. Stress intensity factor, $K$ is usually expressed in the following units:

1. MPa√m for ISO units
2. ksi√in for imperial units
Stress intensity factor, $K$ can be determined using closed form solutions, finite element analysis and a number of other techniques. The solutions relate the remote loading, geometry of the specimen and the crack size to the stress intensity factor, $K$. Using the stress intensity factor in design requires knowledge of the critical stress intensity factor or fracture toughness, $K_C$.

The critical stress intensity factor or fracture toughness, $K_C$ is a mechanical property that measures a material's resistance to fracture. Fracture toughness is used in structural integrity assessment, damage tolerance design, fitness for service evaluation, and residual strength analysis (Xian-Kui et al, 2012). As stated before, $K_C$ is further expressed according to the loading mode, such as $K_{IC}$, $K_{IIC}$, $K_{IIBC}$ for mode I, II and III respectively. When the stress intensity factor reaches the material's structure toughness an existing crack will undergo unstable crack extension (Shukla et al, 2010). Since $K_C$ is material specific its value must be determined for each material of concern. Further, $K_C$ can vary with temperature, component thickness and strain rate.

The critical stress intensity factor, $K_C$ is strongly dependent on plate thickness (Szab & Babuska, 2011). For thin plates it is often the case that the plastic zone around a crack is on the order of the plate thickness. This allows $K_C$ to reach a maximum value ($K_C (max)$). As plate thickness increase, the size of the plastic zone decreases lowering the toughness of the material to some level below $K_C (max)$. As plate thickness continue to increase, the plastic zone size becomes constant and $K_C$ reaches an asymptotic value $K_C (min)$, known as plane strain fracture toughness (Anderson, 2005). This is shown in Figure 2.2.
Besides thickness, the fracture toughness property is analogous to the yield strength property. In tensile test, the material sustain is a stress and will remain elastic until the stress level applied exceeds the yield strength. If yield strength is used as failure criterion, the material fails after the stress level surpasses the yield strength of the material.

For low toughness materials, brittle fracture is the governing failure mechanism and critical stress varies linearly with fracture toughness $K_{IC}$. Figure 2.3 shows the effect of fracture toughness on the governing failure mechanism.

From the figure, it shows that failure will occur when the value of $K = K_{IC}$ where $K$ is the driving force for fracture and $K_{IC}$ is a measure of material resistance.
2.3 Stress Intensity Factor

Stress intensity factors is a measure of the stress field intensity near the tip of an ideal crack in a linear-elastic solid when the crack surfaces are displaced in an opening mode (Xian-Kui et al, 2012). Stress intensity factors can be determined for certain cases if the geometry and remote loading is known. By using a method developed by Westergaard (1930), Irwin (1957) found that the stress and displacement fields in the vicinity of crack tips subjected to the three deformation modes. However, as this research focused on the stresses applied on the plate, the formula for mode I stress intensity factors was given below.

For mode I:

\[
\sigma_x = \frac{K_I}{(2\pi r)^{1/2}} \cos \Theta \left[ 1 - \sin \frac{\Theta}{2} \sin \frac{3\Theta}{2} \right]
\]

\[
\sigma_y = \frac{K_I}{(2\pi r)^{1/2}} \sin \frac{\Theta}{2} \left[ 1 - \cos \frac{\Theta}{2} \sin \frac{3\Theta}{2} \right]
\]

\[
\tau_{xy} = \frac{K_I}{(2\pi r)^{1/2}} \sin \frac{\Theta}{2} \cos \frac{\Theta}{2} \cos \frac{3\Theta}{2}
\]

\[
\sigma = \nu(\sigma_x + \sigma_y)
\]

Based on Erdogan (1965), stress intensity factor of an interface crack is the distribution of stress around an interface crack tip. In the coordinate system in Figure 2.4 the stress formula along the \(x_1\) axis near an interface crack tip is:

\[
\sigma_{x_2} + i\sigma_{x_2} = \frac{K_I + iK_{II}}{\sqrt{2\pi r}} \frac{r''}{l_k}
\]

where \(K_I\) and \(K_{II}\) are the mode I and II stress intensity factors of an interface crack respectively.
The stress intensity factor of the mixed mode crack can be calculated by finite element or boundary element analysis (Aslantas, 2003). Chan et al (1970) stated that the advantage using numerical methods is the calculation is more accurate in terms of near crack tip nodal displacements which is called a displacement correlation method.

Tan & Gao (1990) stated that opening mode $K_I$ and shear mode $K_{II}$ can be defined as:

$$K_I = \sqrt{\frac{2\pi}{L}} D_1 \left[ v^c - 4v^d + 3v^a \right] - D_2 \left[ v^c - 4v^b + 3v^a \right]$$  (2.7)

$$K_{II} = \sqrt{\frac{2\pi}{L}} D_1 \left[ u^c - 4u^d + 3u^a \right] - D_2 \left[ u^c - 4u^b + 3u^a \right]$$  (2.8)

where $L$ is the distance between nodes of $a-c$ or $a-e$. The displacement along the $y$ axis is called $V$ and $D$ is displacement along $x$ axis as shown in Figure 2.5.

Figure 2.5: Joining specimen (a) bi material specimen (b) interface crack at bi material specimen
2.3.1 Stress intensity factors using numerical method

Finite element method is the most commonly used methods for determining stress intensity factor for surface cracks. It is because the high speed of computer and commercial finite element program will make the calculation of stress intensity factor become easier and more possible.

An effective numerical method called the zero element method was proposed for calculating the stress intensity factor in homogenous crack plates. The method then successfully extended to the interfacial crack problems. Both of these methods utilize the stress value at the crack tip computed by finite element method. Figure 2.6 shows the stress intensity factors on bi-material bonded structure.

![Figure 2.6: Geometrical configuration of bi-material bonded plate (Lan et al, 2011)](image)

Previous study by Lan et al (2011) on two dimensional cracks shows that the stress intensity factors was investigated in a bi-material bonded finite strip as shown in Figure 2.7. The approach was by applying the finite element method with varying not only the material combinations but also the relatives crack sizes.
Figure 2.7: The a) shallow and b) deep edge interface cracks in a bonded strip

(Lan et al., 2011)

The bi-material bonded strip in Figure 2.7 is considered with width $W$ and length $2L$. The strip is composed of two elastic, isotropic and homogenous finite strips that are perfect bonded along the interface. Material 1 is the material above the interface and material 2 is the below one. The half length of the strip, $L$ is assumed to be much greater than the width $W$. It is supposed that an edge interface crack with a length of $a$ has initiated at the free edge corner and the strip is subjected to an axial longitudinal uniform tensile stress, $\sigma$.

Table 2.1: Order of stress singularity for various combinations of materials

(Lan et al., 2011)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta = -0.2$</th>
<th>$\beta = -0.1$</th>
<th>$\beta = 0$</th>
<th>$\beta = 0.1$</th>
<th>$\beta = 0.2$</th>
<th>$\beta = 0.3$</th>
<th>$\beta = 0.4$</th>
<th>$\beta = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.05</td>
<td>0.98378</td>
<td>0.99035</td>
<td>0.99800</td>
<td>1.00613</td>
<td>1.01403</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.10</td>
<td>0.96593</td>
<td>0.97774</td>
<td>0.99205</td>
<td>1.00831</td>
<td>1.02512</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.15</td>
<td>0.94684</td>
<td>0.96269</td>
<td>0.98253</td>
<td>1.00626</td>
<td>1.03279</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.20</td>
<td>0.92685</td>
<td>0.94571</td>
<td>0.96987</td>
<td>1.03604</td>
<td>1.07562</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.30</td>
<td>0.90752</td>
<td>0.93713</td>
<td>0.96761</td>
<td>1.02764</td>
<td>1.09640</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.40</td>
<td>0.86549</td>
<td>0.89741</td>
<td>0.94025</td>
<td>1</td>
<td>1.09130</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.50</td>
<td>0.82096</td>
<td>0.85320</td>
<td>0.89662</td>
<td>0.95796</td>
<td>1.05584</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.60</td>
<td>0.77459</td>
<td>0.80597</td>
<td>0.84801</td>
<td>0.90711</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.70</td>
<td>0.75644</td>
<td>0.79606</td>
<td>0.85104</td>
<td>0.93477</td>
<td>1.11741</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.75</td>
<td>0.73090</td>
<td>0.76909</td>
<td>0.82169</td>
<td>0.90048</td>
<td>1.05468</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.80</td>
<td>0.70481</td>
<td>0.74151</td>
<td>0.79163</td>
<td>0.86554</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.85</td>
<td>0.67824</td>
<td>0.71331</td>
<td>0.76091</td>
<td>0.83006</td>
<td>0.94923</td>
<td>1.08125</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.90</td>
<td>0.65105</td>
<td>0.68448</td>
<td>0.72953</td>
<td>0.79410</td>
<td>0.90075</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.95</td>
<td>0.62320</td>
<td>0.65496</td>
<td>0.69745</td>
<td>0.75761</td>
<td>0.85364</td>
<td>0.93488</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1.00</td>
<td>0.59461</td>
<td>0.62466</td>
<td>0.66461</td>
<td>0.72053</td>
<td>0.80731</td>
<td>0.87624</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
The stress intensity factors for the above mentioned problem is plane strain or plane stress are only determined on the two elastic mismatch parameters \( a \) and \( b \) (also known as Dundur’s material composite parameters). The material composite parameters are defined as equations 2.9, 2.10 and 2.11.
The result of $C_1$ and $C_2$ is shown in Table 2.2 and Table 2.3.

Table 2.2 : Values of $C_I$ (Lan et al, 2011)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta = -0.2$</th>
<th>$\beta = -0.1$</th>
<th>$\beta = 0$</th>
<th>$\beta = 0.1$</th>
<th>$\beta = 0.2$</th>
<th>$\beta = 0.3$</th>
<th>$\beta = 0.4$</th>
<th>$\beta = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>1.009</td>
<td>1.074</td>
<td>1.114</td>
<td>1.131</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>0.952</td>
<td>1.034</td>
<td>1.094</td>
<td>1.142</td>
<td>1.163</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.15</td>
<td>0.88</td>
<td>0.991</td>
<td>1.063</td>
<td>1.138</td>
<td>1.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>0.947</td>
<td>1.025</td>
<td>1.119</td>
<td>1.222</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>0.863</td>
<td>0.938</td>
<td>1.047</td>
<td>1.205</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.786</td>
<td>0.852</td>
<td>0.952</td>
<td>1.114</td>
<td>1.485</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.71</td>
<td>0.772</td>
<td>0.857</td>
<td>0.991</td>
<td>1.322</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.60</td>
<td>0.7</td>
<td>0.771</td>
<td>0.872</td>
<td>1.104</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.70</td>
<td>0.635</td>
<td>0.694</td>
<td>0.769</td>
<td>0.919</td>
<td>1.828</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>0.604</td>
<td>0.659</td>
<td>0.723</td>
<td>0.843</td>
<td>1.336</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>0.573</td>
<td>0.626</td>
<td>0.68</td>
<td>0.777</td>
<td>1.087</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td>0.542</td>
<td>0.595</td>
<td>0.64</td>
<td>0.719</td>
<td>0.928</td>
<td>1.558</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.90</td>
<td>0.508</td>
<td>0.565</td>
<td>0.603</td>
<td>0.666</td>
<td>0.815</td>
<td>1.075</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.95</td>
<td>0.46</td>
<td>0.536</td>
<td>0.568</td>
<td>0.619</td>
<td>0.727</td>
<td>0.871</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For three dimensional crack simulation, study of Raju et al (1977) and Leung et al (1995) shows that the stresses will decrease towards the free surface. The SIF value are evaluated by using equations 2.12 and 2.13 as below:

\[
K_I = \lim_{r \to 0} \left[ \ln(\sigma(r)) + \alpha \ln(2\pi r) \right] \quad (2.12)
\]

For engineering purposes, usually SIF are expressed in dimensionless form

\[
K_I = F \sigma \sqrt{\pi a} \quad (2.13)
\]

where \(a\) is the length of the crack, \(\sigma\) is the applied stress and \(F\) is the correction factor depending on the geometry of the crack.

The stress intensity factors are evaluated by equation 2.13 and 2.14 and are shown as Figure 2.9. From figure shown, it is concluded that the stress intensity factors at the specimen surface are much lower than those of the mid plane. The variation of stress intensity factors along the crack front are compared with the plane strain value which was evaluated by integral transform proposed by Gross et al (1964) and are tabulated in Table 2.4.
Figure 2.9 Distribution of stress intensity factors (Leung et al, 1995)

Table 2.4: Stress intensity factors across thickness $a/b = 0.5$, $t/a = 3$ and $v = 0.3$

(Leung et al, 1995)

<table>
<thead>
<tr>
<th>$z/t$</th>
<th>$K/\sigma\sqrt{\pi a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1560</td>
<td>2.8860</td>
</tr>
<tr>
<td>0.3707</td>
<td>2.8344</td>
</tr>
<tr>
<td>0.4515</td>
<td>2.7966</td>
</tr>
<tr>
<td>0.4819</td>
<td>2.6974</td>
</tr>
<tr>
<td>0.4933</td>
<td>2.5326</td>
</tr>
<tr>
<td>0.4976</td>
<td>2.3887</td>
</tr>
<tr>
<td>0.4992</td>
<td>2.1342</td>
</tr>
<tr>
<td>0.4998</td>
<td>2.1607</td>
</tr>
</tbody>
</table>
2.4 Finite Element Analysis

Finite element analysis (FEA) is one of the most powerful and pervasive numerical methods used in modern engineering practices. It was first introduced in 1943 by R. Courant who utilized Ritz method of numerical analysis and minimization of variational calculus to obtain approximate solutions to vibration systems. By early 70’s, FEA was limited to expensive mainframe computers generally owned by the aeronautics, automotive, defense and nuclear industries. However, nowadays rapid progress on the computer technology has made the FEA applicable to all types of parameters.

A central principal of FEA is subdividing the solution domain into smaller, geometrically simple pieces which are called elements, in a process called discretization (Szabo, 2011). Figure 2.10 shows an example of discretization or a mesh of a plate.

![Figure 2.10: FEM model of meshes near crack tip](image)

The finite element method is an approximation of an exact answer and therefore has some amount of error. These errors can come from errors in idealization or discretization as shown in Figure 2.11.
There are generally two types of analysis that are used in industry which is 2-D modeling, and 3-D modeling. While 2-D modeling conserves simplicity and allows the analysis to be run on a relatively normal computer, it tends to yield less accurate results. 3-D modeling, however, produces more accurate results while sacrificing the ability to run on all but the fastest computers effectively. Within each of these modeling schemes, the programmer can insert numerous algorithms (functions) which may make the system behave linearly or non-linearly. Linear systems are far less complex and generally do not take into account plastic deformation. Non-linear systems do account for plastic deformation, and many also are capable of testing a material all the way to fracture.
2.4.1 Basic steps in Finite Element Analysis

The basic steps involved in any finite element analysis consist of the following:

i. Preprocessing phase
   - This phase is used to create and discretize the solution domain into finite elements which is to divide the problems into nodes and elements.
   - Assume a shape function to represent the physical behaviour of an element.
   - Develop equations for an element.
   - Assemble the elements to present the entire problem and construct the global stiffness matrix.
   - Apply boundary conditions, initial conditions and loading

ii. Solution phase
   - This phase is the phase where set of linear or nonlinear algebraic equations will be solved simultaneously to obtain nodal results.

iii. Postprocessing phase
   - Obtain other important informations such as principal stresses, heat fluxes etc.

2.4.2 ANSYS

From the history, ANSYS was released for the first time in 1971. It is a software that is comprehensive general purpose finite element computer program that contains more than 100000 lines of codes. ANSYS is capable of performing static, dynamic, heat transfer, fluid flow and electromagnetism analysis. It has been a leading FEA program for over 30 years.

Nowadays, ANSYS have been used in many engineering fields including aerospace, automotive, electronics and nuclear. Figure 2.12 shows some example of the usage capabilities of ANSYS.
Figure 2.12: Example of ANSYS capabilities (a) ANSYS heat transfer analysis for engine block (b) ANSYS analysis of landing gear simulation (c) electromagnetic analysis of stirring of molten steel in induction furnace (d) seismic analysis and structural optimisation for shopping complex
CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter explains the working procedure to execute the whole projects. This chapter is a step to determine the directions and guidelines to perform the project. It is important as it is to ensure that the project can be completed in time.

3.2 ANSYS Simulation

ANSYS software introduces an effective engineering problem solving through the use of this powerful finite element analysis tool. It is one of the most mature. Widely distributed an popular commercial finite element method programs available. In continuous use and refinement since 1970s, its long history of development has resulted in a code with a vast range of capabilities. For each simulation carry out, the procedure of each simulation is in accordance to the steps in Figure 3.1.
Figure 3.1: Flowchart of the structural analysis by ANSYS
REFERENCES


