FINITE ELEMENT ANALYSIS OF A BEAM STRUCTURE ATTACHED WITH TUNED VIBRATION ABSORBERS

NOOR ASLAMIAH BINTI MAT JUSOH

A thesis submitted in partial fulfilment of the requirements for the award of
Degree of Master of Mechanical Engineering

Faculty of Mechanical and Manufacturing Engineering
Universiti Tun Hussein Onn Malaysia

JANUARY 2015
In this study, the concept of vibration dynamic absorbers used to analyze vibration of a beam structure where simulation has been done by ANSYS APDL. The dynamic vibration absorbers were attached to the fixed-fixed end beam with four different conditions according to its location of placement. The beam modelled by ANSYS was divided into twenty elements. There are twenty one nodes on the beam and distance between nodes is 0.04m. The length of beam, L is 0.8m. The DVA was mounted at node 3 (side of the beam) and node 12 (center of the beam). This research is about the vibration characteristics of the fixed-fixed end beam and dynamic vibration absorbers when placed together and harmonic force, $F_0$ exerted on the beam. The vibration amplitude of the beam structure without DVA and with attached DVAs were compared. The simulation results show amplitude at the natural frequency of beam was decrease close to zero and at the same time amplitude of DVAs were increased. The amplitude of the fixed-fixed end beam and DVAs can be observed significantly at graph were constructed. It is proved that the DVAs absorb vibration at the beam structure. The percentage of reduction for the beam amplitude based on location of DVAs that was placed at the beam.
ABSTRAK

Dalam kajian ini, konsep penyerap getaran dinamik digunakan untuk menganalisis getaran pada struktur rasuk dimana simulasi telah dilakukan menggunakan perisian analisis unsur terhingga (ANSYS APDL). Penyerap getaran dinamik dipasang pada rasuk hujung tetap dengan empat keadaan yang berbeza, berdasarkan lokasi penempatannya. Rasuk dimodelkan menggunakan perisian ANSYS dimana ia telah dibahagikan kepada dua puluh elemen. Terdapat dua puluh satu nod pada rasuk dan jarak antara nod ialah 0.04m. Panjang rasuk, L ialah 0.8m. Penyerap getaran dinamik dipasang pada nod 3 (hujung rasuk) dan nod 12 (tengah rasuk). Kajian ini adalah tentang ciri-ciri getaran pada rasuk hujung tetap dan penyerap getaran dinamik apabila diletakkan bersama dan daya harmonik, $F_0$ dikenakan pada rasuk. Amplitud rasuk tanpa penyerap getaran dinamik dan dipasang dengan penyerap getaran dinamik dibandingkan. Keputusan simulasi menunjukkan amplitud pada frekuansi semulajadi rasuk menurun hampir kepada sifar dan pada masa yang sama amplitud pada penyerap getaran dinamik meningkat. Amplitud pada rasuk hujung tetap dan penyerap getaran dinamik dapat diperhatikan dengan jelas pada graf yang telah dibina. Ini dapat dibuktikan bahawa penyerap getaran dinamik menyerap getaran pada rasuk. Peratus penurunan amplitud rasuk berdasarkan lokasi penyerap getaran dinamik yang diletakkan pada rasuk.
# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TITLE</td>
<td>i</td>
</tr>
<tr>
<td>DECLARATION</td>
<td>ii</td>
</tr>
<tr>
<td>DEDICATION</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>iv</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>v</td>
</tr>
<tr>
<td>ABSTRAK</td>
<td>vi</td>
</tr>
<tr>
<td>CONTENTS</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>x</td>
</tr>
<tr>
<td>LIST OF SYMBOLS AND ABBREVIATION</td>
<td>xiv</td>
</tr>
<tr>
<td>LIST OF APPENDICES</td>
<td>xvi</td>
</tr>
</tbody>
</table>

## CHAPTER 1 INTRODUCTION

1.1 Research background                                                  1
1.2 Problem statement                                                    2
1.3 Research aim                                                          3
1.4 Objectives of the research                                            3
1.5 Scopes                                                                4
1.6 Expected outcomes                                                    4

## CHAPTER 2 LITERATURE REVIEW

2.1 Introduction                                                          6
2.2 Theory of vibration                                                   6
2.3 Control system                                                        9
2.4 Tuned vibration absorber                                              10
2.5 Beam                                                                  13
2.6 Fixed-fixed end beam                                                  14
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Parameters for beam</td>
<td>23</td>
</tr>
<tr>
<td>3.2</td>
<td>Parameters for Dynamic Vibration Absorber (DVA) system</td>
<td>23</td>
</tr>
<tr>
<td>3.3</td>
<td>Natural frequency comparison between values obtained ANSYS and theoretical</td>
<td>29</td>
</tr>
<tr>
<td>4.1</td>
<td>Natural frequency for the fixed-fixed end beam mode shape with attached mass</td>
<td>44</td>
</tr>
<tr>
<td>4.2</td>
<td>Natural frequency for the fixed-fixed end beam mode shape with attached DVAs</td>
<td>49</td>
</tr>
<tr>
<td>4.3</td>
<td>Comparison percentage reduction of the beam amplitude for each condition</td>
<td>63</td>
</tr>
<tr>
<td>4.4</td>
<td>Comparison percentage reduction between result from ANSYS and experiment for beam mounting DVAs for each condition.</td>
<td>63</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

1.1 Effect of the dynamic vibration absorber on the response of machine 2
1.2 Expected amplitude-frequency response of the beam structure 5
1.3 Expected amplitude-frequency response of DVA 1 5
1.4 Expected amplitude-frequency response of DVA 2. 5
2.1 Elementary parts of vibration systems 7
2.2 Displacement, velocity and acceleration in harmonic motion 8
2.3 Specification of vibration levels on a monograph 9
2.4 Two degree of freedom system 10
2.5 Undamped dynamic vibration absorber 11
2.6 Effect of undamped vibration absorber on the response of machine 12
2.7 Graph of displacement as a function of normalized frequency for main mass (beam) and DVA. (a) Main mass displacement, (b) DVA displacement. 13
2.8 Natural frequency and mode shape of a beam fixed at both ends $\omega_n=(\beta_n l)^2(El/Pa)^{1/2}$, $\beta_n l=(2n+1)\pi/14$
2.9 Simplified model 15
3.0 Experiment condition: (a) First mode, (b) Second mode, (c) Third mode and (d) Fourth mode respectively 15
3.1 Simulation process flow chart 22
3.2 Beam illustration 23
3.3 3D view of fixed ends beam without DVA 26
3.4 Fixed ends beam divided by 20 nodes and applied force at node 3. 26
3.5 First mode of fixed ends beam 27
3.6 Second mode of fixed ends beam
3.7 Both DVAs at side
3.8 Finite element model by ANSYS
3.9 Model system for first condition
3.10 First DVA at side and second DVA at beam centre
3.11 Finite element model by ANSYS
3.12 Model system for second condition
3.13 Both DVAs at centre of beam
3.14 Finite element model by ANSYS
3.15 Model system for third condition
3.16 First DVA at centre and second DVA at side.
3.17 Finite element model by ANSYS
3.18 Model system for forth condition
4.1 First mode for fixed ends beam
4.2 Second mode for fixed ends beam
4.3 First mode for fixed ends beam
4.4 Second mode for fixed ends beam
4.5 First mode for fixed ends beam
4.6 Second mode for fixed ends beam
4.7 First mode for fixed ends beam
4.8 Second mode for fixed ends beam
4.9 First mode for fixed ends beam
4.10 Second mode for fixed ends beam
4.11 First mode for fixed ends beam
4.12 Second mode for fixed ends beam
4.13 First mode for fixed ends beam
4.14 Second mode for fixed ends beam
4.15 First mode for fixed ends beam
4.16 Second mode for fixed ends beam
4.17 First mode for fixed ends beam
4.18 Second mode for fixed ends beam
4.19 Response of a beam without DVA to harmonic varying frequency
4.20 Response of a beam attached with DVAs at both side to harmonic varying frequency
4.21 Graph amplitude responses comparison between beam without DVA and beam attached DVAs both at side.  

4.22 (a) Graph amplitude versus frequency for first DVA response at first condition (both DVAs at side).(b) Close view for (a).  

4.23 Graph amplitude versus frequency for second DVA response at first condition (both DVAs at side)  

4.24 Response of a beam attached with First DVA at side and second DVA at beam centre to harmonic varying frequency  

4.25 Graph amplitude responses comparison between beam without DVA and beam attached DVA at side and second DVA at beam centre.  

4.26 Graph amplitude versus frequency for first DVA response at second condition (first DVA at side and second DVA at beam centre).  

4.27 Graph amplitude versus frequency for second DVA response at second condition (first DVA at side and second DVA at beam centre).  

4.28 Response of a beam attached with both DVAs at centre of beam to harmonic varying frequency  

4.29 Graph amplitude responses comparison between beam without DVA and beam attached both DVAs at beam center  

4.30 Graph amplitude versus frequency for first DVA response at third Condition  

4.31 Graph amplitude versus frequency for second DVA response at third condition  

4.32 Response of a beam attached with second DVA at side and first DVA at beam centre to harmonic varying frequency  

4.33 Graph amplitude responses comparison between beam without DVA and beam attached second DVA at side and first DVA at beam centre.  

4.34 Graph amplitude versus frequency for first DVA response at second condition  

4.35 Graph amplitude versus frequency for second DVA response at second condition
LIST OF SYMBOLS AND ABBREVIATIONS

DVA Dynamic Vibration Absorber

FEA Finite Element Analysis

APDL ANSYS Parametric Design Language

DOF Degree of Freedom

f Frequency

c Demper

k Spring stiffness

m Mass

F Force

F0 Harmonic force

x Displacement

ωn Natural frequency

ω Excitation frequency

τ Repetition time

π pi

1D 1 Dimension
2D 2 Dimension
3D 3 Dimension
L Length
b Width
t Thickness
E Young modulus
V Poison ratio
w weight
A Area
ρ Density
I Moment area
V Velocity
LIST OF APPENDICES

<table>
<thead>
<tr>
<th>APPENDIX</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Gantt chart</td>
<td>68</td>
</tr>
<tr>
<td>B</td>
<td>ANSYS Commands</td>
<td>70</td>
</tr>
</tbody>
</table>
INTRODUCTION

1.1 Research background

Every day, most human activity involves vibration in various forms. For example, human can see because of light wave undergo vibration and hear because eardrums vibrate. In engineering field, vibration also considered in design of machines, structures, engines, turbines and control system.

Any motion that repeats itself after an interval of time is called vibration or oscillation [1]. A vibration system includes a way for storing potential energy such as spring or elasticity, a way for storing kinetic energy such as mass or inertia and a way by which energy is gradually lost such as damper. The vibration system involves the transfer of its potential energy to kinetic energy and of kinetic energy to potential energy, alternately [1]. If the system is damped, some energy is lost in each cycle of vibration and must be replaced by an external source if a state is of steady vibration to be maintained.

A tuned vibration absorber is a relatively small spring-mass oscillator that suppresses the response of a relatively large, primary spring-mass oscillator at a particular frequency [2]. Usually, the mass of the dynamic vibration absorber is a few percent of the mass of the primary mass, but the motion of the dynamic vibration
absorber is much greater than the expected motion of the primary mass. The natural frequency of the dynamic vibration absorbers is tuned to be the same as the frequency of excitation. When the excitation frequency is close with the natural frequency, it can be said, the dynamic vibration absorbers are work effectively [2].

![Figure 1.1: Effect of the dynamic vibration absorber on the response of machine [3].](image)

### 1.2 Problem statement

In engineering field, most vibrations are undesirable in structures or machines because will give negative effects such as increased stress, energy losses, induce fatigue, low efficiency and others. Generally, excessive vibration in a system will cause disturbance, discomfort, damage and destruction. So, undesirable vibrations to be reduce to prevent damage on machines or structures. Therefore, this research was undertaken to reduce undesirable vibrations by attaching dynamic vibration absorbers (DVA) to a fixed-fixed end beam structure.

Most of researches focus on the applications of absorbers in the system under harmonic excitations with a single frequency. However, many systems in real applications are excited by multiple frequencies. Because of this reason, this research
being conducted to study dynamic vibration absorbers that can adapt to this situation especially for system having two natural frequencies.

Theoretically, every undamped vibration system can be modelled by an equivalent mass-spring vibration system. A classical DVA consist a single pair of an auxiliary mass-spring system. [4]. Based on experiment done by previous researches, the experiment using two DVAs have been attached in four different types of conditions. The types of condition are:

i) First condition - Both DVAs at side

ii) Second condition - First DVA at side, second DVA at beam centre.

iii) Third condition - Both DVAs at beam centre

iv) Fourth - First DVA at centre, second DVA at side. [4].

But, the result experiment cannot show which one of four different types of condition verify with theory. Therefore, the type of condition is verify with theory will be proved by using FEA.

1.3 Research aim

This research aims to study on absorber system and its tuning for a fixed-fixed end beam.

1.4 Objectives of the research

The objectives of this study are:

i. To determine the vibration characteristics of a vibration beam system attached with Dynamic Vibration Absorber (DVA) by Finite Element Analysis.

ii. To study the vibration characteristics of a dynamic vibration absorber attached on fixed-fixed beam subjected to variable vibration frequency loading

iii. To compare result obtained by FEA (ANSYS) with the experiment done by previous researchers.
1.5 Scopes

To achieve the objectives of the study, the following are the scope/ limitation of project:

i. Using ANSYS Parametric Design Language (APDL) software to obtained the result in this study.

ii. All specimen parameter are identical to the parameter used in experiment done by previous researcher. [4]

iii. Comparison of analysis result between FEA software (ANSYS) and experiment

1.6 Expected outcomes

The expected results of the research are undesirable vibration will absorbed at some frequency range by the Dynamic Vibration Absorber (DVA) that attached on the fixed-fixed beam structure. Therefore, thus helping to reduce vibration on beam and the objectives of research will achieved. Figure 1.2 describes the expected amplitude-frequency response obtained before and after using two DVAs.

When force act on first vibration absorber, the values of $\omega_1$ is higher and $\omega_2$ is lower. Thus, when force act on second vibration absorber, the values of $\omega_1$ is lower and $\omega_2$ is higher. The condition brings to natural frequency. The vibration absorber system consist a spring and a mass interact with one another will resonate at their natural frequency. If system applied energy, it will vibrate at their natural frequency. The level of vibration depends on the energy source and absorption exists in the system. Results from ANSYS will compared and verify result experiment from previous research.
Figure 1.2: Expected amplitude-frequency response of the beam structure.

Figure 1.3: Expected amplitude-frequency response of DVA 1.

Figure 1.4: Expected amplitude-frequency response of DVA 2.
CHAPTER 2

LITERATURE STUDY

2.1 Introduction

In this chapter, present about basic and fundamental theories related the vibration. The following chapter also describe about the dynamic vibration absorbers (DVA) and finite element analysis (ANSYS) that used in research.

2.2 Theory of vibration

Vibration is the motion of particle or a body or system of connected bodies displaced from position of equilibrium [5]. In general, a vibration system consists of a spring, a mass and a damper. The function of spring is a means for storing potential energy, a mass or inertia is a means for storing kinetic energy and a damper is a means by which energy is gradually lost. An undamped vibrating system involves the transfer of its potential energy to kinetic energy and kinetic energy to potential energy, alternatively [1]. Whereas, in a damped vibrating system, some energy is lost in each
cycle of vibrating and should be replaced by an external source if a steady state of vibration is to be maintained.

![Diagram of vibration system](image)

**Figure 2.1: Elementary parts of vibration systems [1]**

When the motion is repeated in equal intervals of time, it is known as periodic motion. Simple harmonic motion is the simplest form of periodic motion. A simple harmonic motion is a reciprocating motion. If $x(t)$ represent the displacement of a vibration of a mass in a vibratory system, the motion can be expressed by the equation

$$X = A \cos \omega t = A \cos \frac{2\pi}{\tau} t$$  \hspace{1cm} (1.1)

Where:

- $A$ is the amplitude of oscillation measured from the equilibrium position of the mass.
- $\tau$ is repetition time. It is called the period of the oscillation.
- $f$ is reciprocal. It is called the frequency, $f = \frac{1}{\tau}$

Any periodic motion satisfies the relationship

$$x(t) = x(t + \tau)$$  \hspace{1cm} (1.2)

That is Period $\tau = \frac{2\pi}{\omega}$ s/cycle

Frequency $f = \frac{1}{\tau} = \frac{\omega}{2\pi}$ cycles/s, or Hz  \hspace{1cm} (1.3)
Where \( \omega \) is called the circular frequency measured in rad/sec.

The velocity and acceleration of harmonic motion are obtained by differentiating eq. (1.1) with respect to time. Using the dot notation for the derivative, obtain

Velocity

\[
\dot{x} = -\omega A \sin \omega t = \omega A \sin (\omega t + \frac{\pi}{2})
\]

(1.4)

Acceleration

\[
\ddot{x} = -\omega^2 A \cos \omega t = +\omega^2 A \cos (\omega t + \pi)
\]

(1.5)

Eqs 1.4 and 1.5 indicate that the velocity and acceleration of a harmonic displacement. Eqs 1.4 and 1.5 are also harmonic of the same frequency but lead the displacement, velocity and acceleration are shown in figure 2 for harmonic motion.

Figure 2.2: a) Displacement in harmonic motion, velocity in harmonic motion and acceleration in harmonic motion [5]. b) Velocity representation
2.3 Control system

The vibration level is an important part before design a vibration suspension system. If a design without knowing what levels of vibration are harmful or at least disagreeable, the vibration suspension systems unable design properly.

Based on vibration monograph as shown in figure below, typical case for which the maximum allowable amplitudes of displacement, velocity and acceleration have been specified. The boundary formed by the line corresponding to these maximum values depends the allowable operating for the system. [6].

![Figure 2.3: Specification of vibration levels on a monograph [6]](image)

Practically, it's possible to reduce the dynamic forces that cause vibration. So, vibration can be controlled on vibration absorbers system because to reduce the vibration on primary system is difficult to change the system. There are several methods can be used to control the vibrations are:

i) Controlling the natural frequency of the system and avoiding resonance under external excitations.
ii) Preventing excessive response of the system, even at resonance by introducing a damping or energy-dissipating mechanism.

iii) Reducing the transmission of the excitation force from one part of the machine to another by the use of vibration absorbers or isolators.

iv) Reducing the response of the system by the addition of an auxiliary mass neutralizer or vibration absorbers [1].

2.4 Tuned vibration absorbers

The vibration absorber also called dynamic vibration absorber (DVA) is a mechanical device used to reduce or eliminate unwanted vibration [1]. Tuned Vibration Absorber is a system which is designed to reduce the amplitude of a Primary System operating at resonance or near to it by attaching an absorber [7].

The basic principle, if operating frequency produced by primary system has undesirable vibration, so, absorber system (secondary system) need to attach at primary system to reducing or eliminating the undesirable vibration so that operating frequency equal to the natural frequency of absorber system. Therefore, the amplitude of primary system can be reduced is zero. The system can be modelled by mass-spring vibration system. Concept of a tuned vibration absorber system attached to the single of freedom system is shown in Figure 2.4 below:

![Figure 2.4: Two degree of freedom system](image)
Where,

\[ k_1 \text{ and } m_1 \text{ represent the primary system} \]

\[ k_2 \text{ and } m_2 \text{ represent the absorber system} \]

\[ F_0 \sin \omega t \text{ is the harmonic force acting on the primary.} \]

The vibration absorber is commonly used in machinery that operates at constant speed because absorber is tuned to one particular frequency and is effective only over a narrow band of frequencies. [1].

If it is acted upon by a force that excitation frequency equivalent with natural frequency of a machine or system, the machine or system may experience excessive vibration. Therefore, the vibration of the machine or system can be reduced by using a vibration neutral or dynamic vibration absorber which is the system consists spring and mass. The dynamic vibration absorber is designed so that the results of natural frequency of system are away from the excitation frequency.
The energy of the main mass is absorbed by the DVA significantly. The motion of the DVA is finite at this resonance frequency, even though there is no damping in either oscillator. This is because the system has changed from a 1-DOF system to a 2-DOF system and now has two resonance frequencies, neither of which equals the original resonance frequency of the main mass and also the absorber. If no damping is present, the response of the 2-DOF system is infinite at these new frequencies. [9]
Figure 2.7: The graph of displacements as a function of normalized frequency for main mass (beam) and DVA [11] (a) Main mass displacement, (b) DVA displacement.

2.5 Beam

Beam is a structure element that able to withstanding load especially by resist bending. The construction a large structure such as building and bridge, beam used as foundation or internal support. In engineering field, a beam have several type of boundary condition such as free-free, fixed-free, fixed-pinned, fixed-fixed and pinned-pinned. Transverse beam vibration, in which the beam vibrates in a direction perpendicular to its length are also called flexural vibration or bending vibration.

The equations of motion of beam are derived according to the Euler-Bernoulli, Rayleigh and Timoshenko theories [8]. The theory of Euler- Bernoulli is neglects the effect of rotary inertia and shear deformation. This theory is applicable
to an analysis of thin beams. While, the Rayleigh theory just considers the effect of rotary inertia and neglect the effect of shear deformation. Though, the Timoshenko theory considers the effects of both rotary inertia and shears deformation and can be used for thick beams.

The equations of motion for the transverse vibration of beams are in the form of fourth-order partial differential equation with two boundary conditions at each end. [8]. The thin beam is considers about the responses of beams under moving loads, beam subjected to axial force, rotating beams, continuous beams and beams on elastic foundation. Based on these three theories, the free vibration solution, the determination natural frequency and mode shapes are considered to use for beam analysis.

2.6 Fixed-fixed ends beam

At the fixed-fixed end beam, the transverse displacement and the slope of the displacement are zero. The first four natural frequency and the corresponding mode shapes are shown in Figure 2.8 below.

\[ \omega_n = (\beta_n l)^2 \left( \frac{E I}{\rho A l^4} \right)^{1/2}, \quad \beta_n l = (2n+1)\pi /2 \]

Figure 2.8: Natural frequency and mode shape of a beam fixed at both ends.
2.7 Concept of multiple vibration absorbers

Illustration of the system that model was simplified equivalent of beam structure shown as Figure 3.2 below, where \( m_M \) is mass and \( k_M \) is elasticity constant. A harmonic force, \( F_0 \) acted on structure of beam and at the structure have large vibration level at forcing frequency, \( \omega^{(n)} \). The number of mode or degree of freedom of the structure represented by \( n \). To suppress the vibration level of the structure up to two vibration modes, two DVAs are attached to the structure. Hypothetically, the first DVA labelled \( m_1 \) and \( k_1 \) will absorb the first mode \( (n = 1) \) of structure vibration and the second DVA labelled \( m_2 \) and \( k_2 \) will absorb the second mode \( (n = 2) \) of structure vibration. [4]

![Figure 2.9: Simplified model [4]](image)

Equations of motion for \( m_M, m_1 \) and \( m_2, \)
\[
m_M \ddot{x}_M + (k_M + k_1 + k_2)x_M - k_1x_1 - k_2x_2 = F_o \sin \omega^{(n)} t
\] (1)
\[
m_1 \ddot{x}_1 - k_1x_M + k_1x_1 = 0
\] (2)
\[
m_2 \ddot{x}_2 - k_2x_M + k_2x_2 = 0
\] (3)
Writing the equations of motion in matrix form to obtain

\[
\begin{bmatrix}
    m_M & 0 & 0 \\
    0 & m_1 & 0 \\
    0 & 0 & m_2
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_M \\
    \ddot{x}_1 \\
    \ddot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
    (k_M + k_1 + k_2) & -k_1 & -k_2 \\
    -k_1 & k_1 & 0 \\
    -k_2 & 0 & k_2
\end{bmatrix}
\begin{bmatrix}
    x_M \\
    x_1 \\
    x_2
\end{bmatrix}
= \begin{bmatrix}
    F_o \sin \omega^{(o)}_t \\
    0 \\
    0
\end{bmatrix}
\] (4)

Writing the matrix equation in terms of maximum displacement amplitude, \( X \) for each respective mass to get

\[
\begin{bmatrix}
    -m_M (\omega^{(o)}_M)^2 & 0 & 0 \\
    0 & -m_1 (\omega_1)^2 & 0 \\
    0 & 0 & -m_2 (\omega_2)^2
\end{bmatrix}
\begin{bmatrix}
    X_M \sin \omega^{(o)}_M t \\
    X_1 \sin \omega_1 t \\
    X_2 \sin \omega_2 t
\end{bmatrix}
+ \begin{bmatrix}
    (k_M + k_1 + k_2) & -k_1 & -k_2 \\
    -k_1 & k_1 & 0 \\
    -k_2 & 0 & k_2
\end{bmatrix}
\begin{bmatrix}
    X_M \sin \omega^{(o)}_M t \\
    X_1 \sin \omega_1 t \\
    X_2 \sin \omega_2 t
\end{bmatrix}
= \begin{bmatrix}
    F_o \sin \omega^{(o)}_t \\
    0 \\
    0
\end{bmatrix}
\] (5)

Where \( \omega^{(o)}_M \) is the natural frequency of the main structure mass, \( \omega_1 \) and \( \omega_2 \) are the natural frequency of the DVA tuned to the first and second natural frequency of the main structure mass respectively and \( n \) is the number of mode for the main structure \((n = 1, 2, \ldots)\).

Assuming the solution are harmonic. The matrix equation becomes

\[
\begin{bmatrix}
    (k_M + k_1 + k_2) - m_M (\omega^{(o)}_M)^2 & -k_1 & -k_2 \\
    -k_1 & k_1 - m_1 (\omega_1)^2 & 0 \\
    -k_2 & 0 & k_2 - m_2 (\omega_2)^2
\end{bmatrix}
\begin{bmatrix}
    X_M \sin \omega^{(o)}_M t \\
    X_1 \sin \omega_1 t \\
    X_2 \sin \omega_2 t
\end{bmatrix}
= \begin{bmatrix}
    F_o \sin \omega^{(o)}_t \\
    0 \\
    0
\end{bmatrix}
\] (6)

\( \omega^{(i)} = \omega^{(o)}_M \)

The forcing frequency \( \omega^{(o)} \) operates at the first natural frequency of the main mass.

Expanding the first row of Eqn. [6] to solve for \( X_M \)

\[
\left[(k_M + k_1 + k_2) - m_M (\omega^{(o)}_M)^2\right] X_M \sin \omega^{(o)}_M t - k_1 X_1 \sin \omega_1 t - k_2 X_2 \sin \omega_2 t = F_o \sin \omega^{(o)}_M t
\] (7)

Expanding the second row of Eqn. [6] to solve for \( X_1 \)

(8)
\[-k_1X_M \sin \omega_M^{(1)} t + [k_1 - m_1(\omega_1)^2]X_1 \sin \omega_1 t = 0\]

\[X_1 = \frac{k_1X_M \sin \omega_M^{(1)} t}{[k_1 - m_1(\omega_1)^2]\sin \omega_1 t}\]  \hspace{1cm} (9)

Expanding the third row of Eqn. (6) to solve for \(X_2\)

\[-k_2X_M \sin \omega_M^{(1)} t + [k_2 - m_2(\omega_2)^2]X_2 \sin \omega_2 t = 0\]  \hspace{1cm} (10)

\[X_2 = \frac{k_2X_M \sin \omega_M^{(1)} t}{[k_2 - m_2(\omega_2)^2]\sin \omega_2 t}\]  \hspace{1cm} (11)

From Eqn. (9) and (11), it can be seen that when \(X_M\) becomes zero, \(X_1\) and \(X_2\) are also become zero. However, both \(X_1\) and \(X_2\) cannot equal to zero at the same time because one of the DVA has to absorb the vibration of the main mass. In this case, let say \(X_2\) becomes zero so that \(m_1\) absorbs \(m_M\) vibrations. In order to make \(m_1\) absorbs \(m_M\) vibrations, \(\omega_1\) must be tuned to \(\omega_M^{(1)}\) or vice versa.

Case 1

In this case, let say \(X_2\) becomes zero that \(m_1\) absorbs \(m_M\) vibrations. In order to make \(m_1\) absorbs \(m_M\) vibration \(\omega_1\) must be tuned to \(\omega_M^{(1)}\).

Therefore, Eqn. (7) becomes

\[\left[ (k_M + k_1 + k_2) - m_1(\omega_1)^2 \right]X_M \sin \omega_M^{(1)} t - k_1X_1 \sin \omega_1 t = F_o \sin \omega_M^{(1)} t\]  \hspace{1cm} (12)

Substituting Eqn. [9] into [12], tuned \(\omega_1\) to \(\omega_M^{(1)}\) and rearranging to obtain

\[X_M = \frac{F_o \left[ k_1 - m_1 (\omega_1)^2 \right]}{[k_M + k_1 + k_2] - m_1 (\omega_M^{(1)})^2 \left[ k_1 - m_1 (\omega_M^{(1)})^2 \right] - k_1^2}\]  \hspace{1cm} (13)

To make \(X_M\) equals to zero, the numerator of Eqn. (13) must equals to zero, therefore

\[\omega_M^{(1)} = \sqrt{\frac{k_1}{m_1}} = \omega_1\]  \hspace{1cm} (14)

Case 2:
In this case, let say \( X_1 \) becomes zero that \( m_2 \) absorbs \( m_M \) vibrations. In order to make \( m_2 \) absorbs \( m_M \) vibration \( \omega_2 \) must be tuned to \( \omega_M^{(2)} \)

Therefore, Eqn. [7] becomes

\[
\left( k_M + k_1 + k_2 \right) - m_2 \left( \omega_M^{(2)} \right)^2 X_M \sin \omega_M^{(2)} t - k_1 X_1 \sin \omega_1 t - k_2 X_2 \sin \omega_2 t = F_o \sin \omega_M^{(2)} t
\]

(15)

Substituting Eqn. (11) into (15), tuned \( \omega_1 \) to \( \omega_M^{(2)} \) and rearranging to obtain

\[
X_M = \frac{F_o \left[ k_2 - m_2 \left( \omega_M^{(2)} \right)^2 \right]}{\left( k_M + k_1 + k_2 \right) - m_2 \left( \omega_M^{(2)} \right)^2 \left[ k_2 - m_2 \left( \omega_M^{(2)} \right)^2 \right] - k_2^2}
\]

(16)

Therefore,

\[
\omega_M^{(2)} = \sqrt{\frac{k_2}{m_2}} = \omega_2
\]

(17)

Referring to Eqn. (14) and Eqn. (17), it have been proved that the vibration level for each mode or degree of freedom can be suppress by tuning each of the DVA to appropriate mode accordingly. This method also feasible for the systems with higher degree of freedoms. [4]

2.8 Previous researches

Based on previous study, there are many researches about dynamic vibration absorber (DVA). Zainulabidin et. al. (2012) has studied about transverse vibration of a fixed-fixed end beam attached with dynamic vibration absorbers (DVA). The counter back motion used as concept of DVA for eliminating the vibration on beam.

The concept of DVA is eliminating the vibration by a counter back motion. A new control strategy has been tested experimentally in order to absorb vibration of a beam structure. Only the first two natural modes will tested and the beam structure is assumed to vibrate in transverse directly only. The experiment using two DVAs has been attached in four different conditions. The types of condition are:

v) First condition- Both DVAs at side

vi) Second condition- First DVA at side, second DVA at beam centre.
vii) Third condition – both DVAs at beam centre

viii) Fourth – first DVA at centre, second DVA at side. [4].

Figure 2.10: Experiment condition: (a) First mode, (b) Second mode, (c) Third mode and (d) Fourth mode respectively [4]

The results of experiment show that, DVAs have successfully absorbed the beam vibration and thus helping to reduce the vibration amplitude of the beam.

While, Alva et. al. (2011) used ANSYS Parametric Design Language (APDL) as finite element analysis to determine results compared with experiment result. In this research, used SOLID 92 for element type and mass element that used was STRUCTURAL MASS 3D. Primary system represented by machine or a system and secondary system represent by vibration absorber system.

At the fundamental mode of vibration both the systems move in the same direction and at the second mode both the systems vibrate out of phase which is proven using ANSYS. The harmonic analysis is used to study the system response at various operating frequencies ANSYS.

Harmonic force with different frequency between two limits is chosen and the system response to these frequencies can be studied. Harmonic analysis clearly shown the safe operating limit which is proven by amplitude levels of resonating frequencies. By introduction of the tuned mass absorber the primary system’s first
mode natural frequency is changed by a small unit. The harmonic response of primary system and secondary system clearly showing the change in systems natural frequency after the introduction of tuned mass absorber.
CHAPTER 3

METHODOLOGY

3.1 Introduction

This chapter describes about the methodology in this research. The flow chart as shown in Figure 3.1 presents overall research activity. Gantt chart (APPENDIX A) shows the activity planning of research. Previous researches referred to determine the method of study for solving the problems. Simulation beam structure attached with tuned vibration absorbers performed using ANSYS (Finite Element Software). All specimen parameter of study are identical to the parameter used in experiment done by Zainulabidin et. al. (2013).
3.2 Methodology flow chart

![Methodology flow chart diagram]

Start

- Draw Model beam tune with vibration absorbers using finite element software

Meshing Model ANSYS APDL - Mechanical

Condition parameters selection

Simulation using ANSYS (APDL)

Result analysis

Yes → Discussion and conclusion

End

No

Figure 3.1: Methodology flow chart
3.3 Parameters Selection

All parameters experiment for beam and vibration absorbers system as shown in tables below. Parameters for fixed-fixed end beam shown in Table 3.1 and material used for beam is aluminium. Table 3.2 shows parameters for DVA system. The right sequences of the steps or procedures will lead to easier and smoother of experiment. Besides that, the parameters and units identification in this simulation are very important to obtain the accurate data and values.

![Figure 3.2: Beam illustration](image)

Table 3.1: Parameters for beam

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Geometric properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young modulus, $E = 69$GPa</td>
<td>Length, $L = 0.8m$</td>
</tr>
<tr>
<td>Poisson ratio, $\nu = 0.35$</td>
<td>Width, $b = 0.025m$</td>
</tr>
<tr>
<td>Density, $\rho = 2800$ kg/m$^3$</td>
<td>Thickness, $t = 0.002m$</td>
</tr>
<tr>
<td>Weight, $w = 0.118$ kg</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Parameters for Dynamic Vibration Absorber (DVA) system

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>First mass, $m_1 = 0.1kg$</td>
<td>Harmonic force, $F_0 = 29.42$ kgm/s$^2$</td>
</tr>
<tr>
<td>Second mass, $m_2 = 0.1kg$</td>
<td></td>
</tr>
<tr>
<td>First spring stiffness, $k_1 = 973$ N/m</td>
<td></td>
</tr>
<tr>
<td>Second spring stiffness, $k_2 = 7713$ N/m</td>
<td></td>
</tr>
</tbody>
</table>
The formula to obtain the suitable spring stiffness are as followed:

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

So, the spring stiffness, \( k = (2\pi f)^2 m \)

Where,

\( m \) is mass absorber

\( f \) is frequency of beam

### 3.4 Vibration absorbers by ANSYS

Dynamic analysis in ANSYS APDL 14.0, the term used to describe the calculation of eigenvector and eigenvalues is namely mode extraction. There are several mode extraction method are available in ANSYS such as Block Lanczos (default), Subspace, Power Dynamics, Reduced, Unsymmetric, Damped (full) and QR Damped. The Method selected depends on model size and the particular application. In this research, model analysis was conducted on the structure using Block Lanczos as mode extraction method for solving eigenvalues.

### 3.5 Element selection

There are many elements in ANSYS software. Elements selected based on the type of the analysis study. In this research, using three elements are;

i) **COMBIN14**

COMBIN14 has longitudinal or torsional capability in 1-D, 2-D, or 3-D applications [10]. The degree of freedom for longitudinal spring-damper option is up to three at each node and uniaxial with tension-compression element. Translations of the element in the nodal x, y, and z directions and not consider about bending or torsion. The option of torsional spring-damper is a purely rotational element. It is with three degree of freedom for
REFERENCES


While the tuned Vibration Absorber is a system which is designed to reduce the amplitude of a Primary System operating at resonance or near to it by attaching an absorber Hill. 1956


[18] Anthony C. Webster, & Rimas Vaicaitis. Application of tuned mass dampers to control vibrations of composite floor systems. Engineering